s-prof(lucy) • ∀x ( is-prof(x) ⇒ is-person(x) ) • is-dean(fuchs) • ∀x (is-dean(x) ⇒ is-prof(x)) • ∀x (∀y ( is-prof(x) ∧ is-dean(y) ⇒ is-friend-of(y,x) ∨ ¬knows(x, y) ) ) • ∀x (∃y ( is-friend-of (y, x) ) ) • ∀x (∀y (is-person(x) ∧ is-person(y) ∧ criticize (x,y) ⇒ is-friend-of (y,x))) • criticize(lucy,fuchs) Question: Is Fuchs is friend of Lucy? is-friend-of(fuchs,lucy)

Based on the given premises, we can reason as follows:

1. According to the first premise, any professional is a person. Therefore, Lucy is a person.
2. The second premise states that Fuchs is a dean, and according to the third premise, any dean is also a professional. Therefore, Fuchs is a professional.
3. The third premise states that if someone is a professional and someone else is a dean, then they are either friends or the professional does not know the dean. Since Fuchs is a dean and Lucy is a professional, this premise applies to them. However, we do not know if they are friends or not.
4. The fourth premise states that everyone has at least one friend who is a friend of theirs. Therefore, if Lucy has a friend, that friend is a friend of Lucy's.
5. The fifth premise states that if person X criticizes person Y, then Y is a friend of X. Since Lucy criticizes Fuchs, and we know from premises 1 and 2 that they are both persons, this premise applies to them. Therefore, Fuchs is a friend of Lucy.

Based on this reasoning, we can conclude that Fuchs is a friend of Lucy. Therefore, the statement "is-friend-of(fuchs,lucy)" is true.

Lucy\* is a professor 2. All professors are people. 3. John is the dean. 4. Deans are professors. 5. All professors consider the dean a friend or don’t know him. 6. Everyone is a friend of someone. 7. People only criticize people that are not their friends. 8. Lucy criticized John .

Based on the given premises, we can reason as follows:

1. Lucy is a professor, according to premise 1.
2. All professors are people, according to premise 2. Therefore, Lucy is a person.
3. John is the dean, according to premise 3.
4. Deans are professors, according to premise 4. Therefore, John is also a professor.
5. All professors consider the dean a friend or don't know him, according to premise 5. Therefore, either Lucy considers John a friend, or she does not know him.

Lucy\* is a professor 2. All professors are people. 3. John is the dean. 4. Deans are professors. 5. All professors consider the dean a friend or don’t know him. 6. Everyone is a friend of someone. 7. People only criticize people that are not their friends. 8. Lucy criticized John .Question: Is John no friend of Lucy? ¬is-friend-of(John ,lucy)

Based on the given premises, we can reason as follows:

1. Lucy is a professor, according to premise 1.
2. All professors are people, according to premise 2. Therefore, Lucy is a person.
3. John is the dean, according to premise 3.
4. Deans are professors, according to premise 4. Therefore, John is also a professor.
5. All professors consider the dean a friend or don't know him, according to premise 5. Therefore, either Lucy considers John a friend, or she does not know him.
6. Everyone is a friend of someone, according to premise 6. Therefore, John is a friend of someone, unless he is the exception to this rule.
7. People only criticize people that are not their friends, according to premise 7. Therefore, if Lucy criticized John, it implies that she is not his friend.
8. Premise 8 states that Lucy criticized John.

Based on this reasoning, we can conclude that John is not a friend of Lucy's. Therefore, the statement "¬is-friend-of(John, Lucy)" is true.

Can this be proven using the concept of Resolution

Yes, we can use the Resolution principle to prove that John is not a friend of Lucy's. We can start by assuming the negation of the conclusion we want to reach, i.e., assume that John is a friend of Lucy's:

Assumption: is-friend-of(John, Lucy)

Using premise 5, we know that all professors either consider the dean a friend or don't know him. Since John is a professor and the dean, he must either consider himself a friend of Lucy or not know her.

Case 1: John considers himself a friend of Lucy If John is a friend of Lucy, then he and Lucy are friends of each other, which contradicts premise 7 that states people only criticize people that are not their friends. Therefore, we can derive the following clause from the assumption and premise 7:

¬is-friend-of(John, Lucy) ∨ ¬criticize(Lucy, John)

Case 2: John does not know Lucy If John does not know Lucy, then he cannot be her friend, which means the assumption contradicts premise 5. Therefore, we can derive the following clause from the assumption and premise 5:

¬is-friend-of(John, Lucy) ∨ ¬¬knows(John, Lucy)

By resolving the two derived clauses above, we get:

¬criticize(Lucy, John) ∨ knows(John, Lucy)

However, premise 8 states that Lucy criticized John, which means ¬criticize(Lucy, John) is true. Therefore, we can conclude that knows(John, Lucy) is true. This contradicts the derived clause above, which means the assumption is false:

¬is-friend-of(John, Lucy)

Therefore, we have proven that John is not a friend of Lucy's.

Prove the same using concept of backward chaining

We can use backward chaining to prove that John is not a friend of Lucy's:

Goal: ¬is-friend-of(John, Lucy)

1. From premise 5, we know that all professors consider the dean a friend or don't know him. Since John is the dean, he must either consider himself a friend of Lucy or not know her.
2. If John considers himself a friend of Lucy, then he and Lucy are friends of each other, which contradicts premise 7 that states people only criticize people that are not their friends. Therefore, we can infer that John does not consider himself a friend of Lucy.
3. From premise 8, we know that Lucy criticized John. Since people only criticize people that are not their friends, we can infer that Lucy is not a friend of John's.
4. If John is not a friend of Lucy's, then ¬is-friend-of(John, Lucy) is true.

Therefore, we have proven that John is not a friend of Lucy's using backward chaining.